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MECHANISMS OF VISCOELASTIC SOIL DEFORMATION UNDER CYLINDER ROLLING

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The mechanisms of deformation by a circular rolling cylinder of a viscoelastic soil with depth-varying physical properties have been investigated. A method for calculating the indices characterizing the rheological properties, the stressed-strained state, and the density of the soil upon the first passage over the soil and upon many passages of a cylinder over one track is proposed. A quantitative assessment of the influence of the basic factors on the change in the rheological properties of the soil and their tending to elastic ones as a result of a number of cylinder passages over one track has been made.

One basic factor of anthropogenic actions on soil is its loading as vehicles and farm machinery pass over it. Modern farming technologies require multiple passages of mobile machines over a field. To develop measures for preventing supercompaction of soils and creating favorable conditions for agricultural plants, it is essential to elucidate the laws of change in the properties of the soil as a result of its deformation by the wheels of vehicles and rollers.

This problem can be solved on the basis of investigations on rolling friction. Reviews of some of the results that have been obtained may be found in [1, 2]. In [3–5], problems on the cylinder rolling on a viscoelastic and a plastic base were considered. As in many other works, here the initial properties of the base at different depths were taken to be constant; the contact arc is considered to be small compared to the cylinder radius. An important contribution to the theory of wheel rolling with the formation of a deep track was made in [6–8]. The present paper gives the results of theoretical and experimental investigations of the rolling of circular cylinders on a compacting soil, including rollers and wheels of mobile machines as they pass over one track. Unlike in [3–5], we consider the contact surface as part of a cylindrical surface rather than as part of a plane. It is taken into account that the soil compactness and rheological properties vary with depth of its deforming layer.

At moisture contents w lower than the total moisture capacity under the action of a load the soil is compacted and hardened. At such w noncompacted soils exhibit viscoelastic properties [9–15].

On the basis of a theoretical analysis of the experimentally elucidated mechanisms of deformation of compactible soils [9, 10], the author suggested [11] modeling their rheological properties by the differential equation

$$\sigma_t' + p\sigma = q\varepsilon_t' . \tag{1}$$

The limits of applicability of Eq. (1) for particular soils are determined experimentally. For instance, its suitability for a sod-podzol slightly loamy soil of a certain granulometric composition at w = 16–26% and at certain values of t, σ, and ε [11–14], peat-marshy soil [14], and black earths of the Middle Volga Basin [15] has been confirmed. The advantages of using Eq. (1) have been shown. Under deformation by the harmonic law $p = \omega g$. Linear correlation dependences of g and q of the investigated sod-podzol soil on ρ, w, and ω were found [13, 14].

Consider a rolling with slipping (positive or negative (i.e., sliding)) of an elastic circular cylinder on a soil whose viscoelastic properties are modeled by Eq. (1). The axial velocity v_{ax} of the cylinder and its angular velocity ω are taken to be constant and the soil surface — horizontal. The slipping coefficient of the cylinder $\delta = 1 - v_{ax}/\omega R$. The cylinder is subjected to the action of the following forces applied to its axis: vertical G and horizontal F forces, the torque M, and the reaction forces of the soil. The latter forces distributed over the contact surface have been replaced by the resultant forces — vertical N and horizontal T (Fig. 1). The values of N and T depend on

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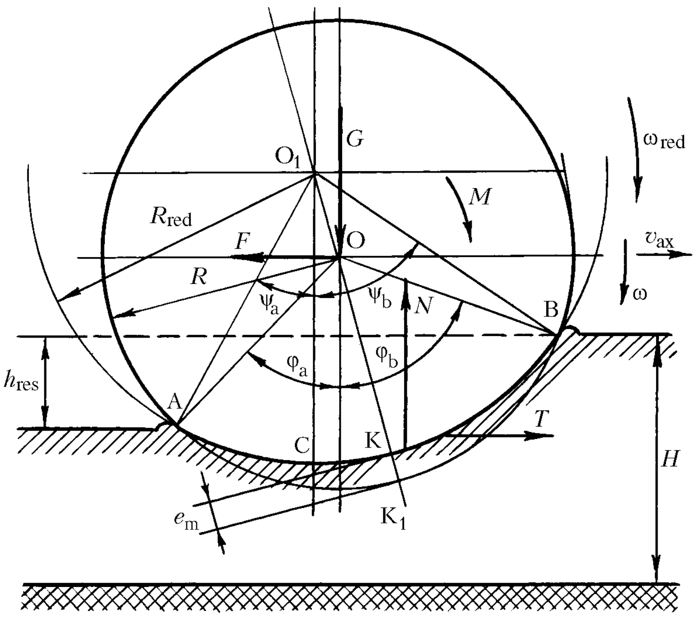

Fig. 1. Scheme of the interaction between the driving circular cylinder and the soil.

the viscoelastic properties of the soil and the sliding friction forces between the cylinder and the soil. In the driving cylinder, the directions of M and ω coincide, $\delta > 0$, and the reaction T is directed towards the axis motion. For this case, we took $T > 0$ and the driving torque $M > 0$. In the driven cylinder, the directions of M and ω are opposite, $\delta < 0$, $T < 0$, and the braking torque $M < 0$.

On the basis of the results of statistical processing of experimental data, the dependence of the density ρ of the upper deforming layer of the soil before its compaction on the depth y was taken in the form of two linear portions:

$$\rho(y) = \begin{cases} \rho_{01} + k_1 y & \text{at } y \in [0, H_1]\,; \\ \rho_{02} + k_2 y & \text{at } y \in (H_1, H]\,, \end{cases} \tag{2}$$

where $\rho_{02} = \rho_{01} + (k_1 - k_2)H_1$.

Since ρ is described by expression (2), we have the linear dependences $q = q(y)$ and $p = p(y)$ at $y \in [0, H_1]$ and $y \in (H_1, H)$.

The cylinder length L was assumed to be fairly large. Therefore, the soil deformation by its rolling is approximately plane. The length of the line of contact of an elastic cylinder of radius R with the soil depends on the contact angles $\varphi_b > 0$ and $\varphi_a < 0$. The line of contact with the soil of the elastic cylinder in its central longitudinal section is approximated by the arc AKB of the circle of a conventional rigid cylinder of radius R_{red}. The length of this arc is determined by the values of R_{red} and $\psi_b > 0$ and $\psi_a < 0$ (Fig. 1). The angle ψ_b is equal to the angle CO_1B, and the angle ψ_a — to the angle AO_1C.

On the contact line, the soil shifts at the contact surface points and, consequently, the deformations and the contact stresses are functions of the variable t or the angle $\psi = \psi_b - \omega_{red} t$ ($\psi \in [\psi_a, \psi_b]$). At $\psi \in [0, \psi_b]$ the soil is compressed in the vertical direction and shifted in the horizontal direction. At $\psi \in [\psi_a, 0)$ reversible deformations of the soil occur, $\psi_b > |\psi_a|$.

At the contact line points

$$\varepsilon(\psi) = h(\psi)/H_p = R_{red}(\cos\psi - \cos\psi_b)/H_p\,. \tag{3}$$

Using (1) and (3) and the boundary conditions $\sigma(\psi_a) = 0$ and $\sigma(\psi_b) = 0$, we obtain the formula for determining the soil compression stresses on the contact line

$$\sigma(\psi) = \frac{qR_{\mathrm{red}}}{H_{\mathrm{p}}(g^2+1)} [\cos\psi + g\sin\psi - (\cos\psi_{\mathrm{b}} + g\sin\psi_{\mathrm{b}}) \exp(-g(\psi_{\mathrm{b}} - \psi))] \tag{4}$$

and the equation relating ψ_{b} and ψ_{a}:

$$\exp(-g\psi_{\mathrm{b}})(\cos\psi_{\mathrm{b}} + g\sin\psi_{\mathrm{b}}) - \exp(-g\psi_{\mathrm{a}})(\cos\psi_{\mathrm{a}} + g\sin\psi_{\mathrm{a}}) = 0\,. \tag{5}$$

Under deformation of a viscoelastic medium, including soil, a phase shift of deformations and stresses is observed: their maxima points do not coincide. In the case of soil deformation by a rolling cylinder, the maximum of its deformation corresponding to the points on the contact line is attained at $\psi = 0$. The angle ψ_{m} at which the compression stresses take on the maximum value of σ_{m} is determined from the equation

$$g\cos\psi_{\mathrm{m}} - \sin\psi_{\mathrm{m}} - g(\cos\psi_{\mathrm{b}} + g\sin\psi_{\mathrm{b}}) \exp(-g(\psi_{\mathrm{b}} - \psi_{\mathrm{m}})) = 0\,, \tag{6}$$

obtained under the condition $\sigma'(\psi)\big|_{\psi=\psi_{\mathrm{m}}} = 0$.

When a cylinder is rolling in a plane perpendicular to its axis and passing through the axis center, in the soil a plane deformation wave propagates. This wave consists of a compression wave caused by the vertical shifts $v(y, t)$ of the soil and a shear wave caused by its horizontal shifts.

The soil compression under the action of the stress σ is described by the equation of motion

$$\sigma'_y = \rho(y)\, v''_{t^2}\,. \tag{7}$$

Based on Eqs. (1) and (7), we obtained a differential equation with fourth-order partial derivatives and variable coefficients modeling the compression-wave propagation in a viscoelastic soil whose density linearly depends on the depth:

$$\rho(y)\, v^{\mathrm{IV}}_{t^3y} - q(y)\, v^{\mathrm{IV}}_{ty^3} + k_i v'''_{t^3} + p(y)\,\rho(y)\, v'''_{t^2y} - 2b_i v'''_{ty^2} + (2\tilde{c}_i\rho(y) + k_i p(y))\, v''_{t^2} = 0\,. \tag{8}$$

At $k_i = b_i = \tilde{c}_i = 0$ Eq. (8) transforms into the wave equation for a viscoelastic soil of constant density analogous in its structure to the wave equation for the Maxwell medium [16]. Let us represent the mathematical model of the problem of determining the vertical shifts of the soil as a result of the cylinder rolling as the problems of finding $v(y, t)$ at: 1) $t \in [0, t_1]$; 2) $t \in [t_1, \infty)$. We first assume that the region of soil-deformation propagation is not bounded below, i.e., $H \to \infty$.

Problem 1 is formulated as follows: find a solution of Eq. (8) satisfying the boundary conditions

$$\left.\begin{aligned} v(0, t) &= R_{\mathrm{red}}(\sin(\alpha_0 + \omega_{\mathrm{red}}t) - \sin\alpha_0)\,,\\ v(\infty, t) &= 0\,, \end{aligned}\right\} \quad t \in [0, t_1]\,, \tag{9}$$

and the initial conditions

$$\left.\begin{aligned} v(y, 0) &= 0\,,\\ v'_t(y, 0) &= 0\,,\\ v''_{t^2}(y, 0) &= 0\,, \end{aligned}\right\} \quad y \in [0, \infty)\,, \tag{10}$$

where $\alpha_0 = \pi/2 - \psi_{\mathrm{b}}$; $t_1 = (\psi_{\mathrm{b}} + |\psi_{\mathrm{a}}|)/\omega_{\mathrm{red}}$.

The solution of the boundary-value problem (8)–(10) was sought on the basis of the results of [16, 17]. The approximate solution at $t \in [0, t_1]$ was obtained in the form

$$v(y, t) = R_{\mathrm{red}} \exp(-r_1(y)) [\sin(\alpha_0 + \omega_{\mathrm{red}}t - r_2(y)) - \sin\alpha_0]\,. \tag{11}$$

The sought functions $r_1(y)$ and $r_2(y)$ characterizing the compression-wave damping with depth were determined approximately by the collocation method of [18].

In the formulas found, the unknowns are ψ_b, ψ_a, R_{red}, and $H_{t.p}$. They were determined as the solution of the system of four nonlinear equations with four unknowns and with variable coefficients obtained in the present paper. This system incorporates: Eq. (5), the equation reflecting the equality to each other of the values of the maximum radial deformation of the elastic cylinder found from the physical and geometric prerequisites, and two equations obtained from the conditions $N = G$ and $v(H_{t.p}, t_1) = 0$. In the given domains of variability of the unknowns, this system has a unique solution.

If the solution of the system yields $H_{t.p} \le H$, then $H_p = H_{t.p}$. If $H_{t.p} > H$, this means that the region of soil-deformation propagation is bounded below by the value of H and $H_p = H$. The boundary $y = H$ excites the reflected wave [19]. The expression for determining $v(y, t)$ in this case has been found. We find ψ_b, ψ_a, and R_{red} thereby as the solution of a certain system of three nonlinear equations in three unknowns.

The solution of (11) and the obtained numerical values of ψ_b, ψ_a, R_{red}, and H_p were used to form the boundary and initial conditions of *Problem* 2, formulated as follows: find a solution of Eq. (8) satisfying the boundary conditions

$$\left.\begin{array}{r} v\,(0, t) = h_{res}\,, \\ v\,(H_p, t) = 0\,, \end{array}\right\} \quad t \in (t_1, \infty)\,, \tag{12}$$

and the initial conditions

$$\left.\begin{array}{ll} v\,(y, t_1) = \tilde{\varphi}_1\,(y)\,, & v'_t\,(y, t_1) = \tilde{\varphi}_2\,(y)\,, \\ v''_{t^2}\,(y, t_1) = \tilde{\varphi}_3\,(y)\,, & \end{array}\right\} \quad y \in [0, H_p]\,. \tag{13}$$

The functions $\tilde{\varphi}_1(y)$, $\tilde{\varphi}_2(y)$, and $\tilde{\varphi}_3(y)$ were determined from (11) at $t = t_1$:

$$h_{res} = R_{red}\,(\cos\psi_a - \cos\psi_b)\,. \tag{14}$$

The approximate solution of the boundary-valued problem (8), (12), (13) was obtained by using the results from [19], the Laplace–Carson transform, and the collocation method [18] in the form

$$v\,(y, t) = \frac{h_{res}\,(H_p - y)}{H_p} + \sum_{j=1}^{s} C_j\,(t)\sin\frac{\pi j}{H_p}\,y\,. \tag{15}$$

At $t \to \infty$ we obtain $C_j(t) \to \tilde{C}_j$, $v(y, t) \to v(y, \infty) = v_{st}(y)$. If the boundary $y = H$ excites the reflected wave, then $\tilde{C}_j \approx 0$ and $v_{st}(y)$ are determined approximately by the first term in the right-hand formula of (15).

The soil-density increment $\Delta\rho$ was obtained on the basis of solving the problem on the soil shifts at various depths. At $\tilde{C}_j \approx 0$ (which corresponds to the results of the calculations performed) and $H_p \le H_1$,

$$\Delta\rho\,(y + v_{st}\,(y)) \approx \frac{h_{res}\,(2\rho_{01} + k_1 h_{res})\,(H_p - y)}{(H_p + \mu h_{res})^2}\,. \tag{16}$$

The soil density upon a cylinder passage is approximately characterized by a dependence of the form (2) but with changed values of the parameters.

As a result of solving the problem on the rolling of a cylinder with $\delta > 0$ and $\delta < 0$ over a viscoelastic soil, we also obtained, apart from formulas (3)–(6), (14), and (16), relations for determining the force $F = T$, the torque M, etc.

Assuming in the relations found for the rigid cylinder $R_{red} = R$ and $\omega_{red} = \omega$, we obtain the formulas and algorithms for determining the characteristics of the rigid cylinder–soil interaction.

As a result of the theoretical study made, we propose a method for calculating the indices characterizing the viscoelastic properties, the stressed-strained state, and the compactness of the soil upon the first passage of a rolling circular cylinder over the soil and upon a number of passages over one track at stresses $\sigma < \sigma_{str}$. The soil compactness upon the first passage of the cylinder represents the initial compactness before its second passage. For the second and the following passages of the cylinder over the track made, calculations analogous to those for the first passage but with changed values of the quantities g and q, as well of the parameters of the dependence of the form (2) should be made.

According to the large amount of experimental data, under rolling of automobile and tractor wheels over compacting cohesive soils, the most significant components of base deformations are the vertical compression and the horizontal shift. Soil deformations along the wheel axis are very small [8]. In this connection, the deformation of cohesive soils under rolling of automobile and tractor wheels can approximately be assumed to be plane. Therefore, the proposed computing method can be used to determine the indices of the interaction between the wheels of mobile machines and compressing cohesive soils. The assumption that the base deformation under rolling of wheels is approximately plane was made in [6–8] and in many other works. The theoretical results obtained therein agree with the experimental data of [8].

A computational procedure for the above indices and computer programs for its realization have been developed. These programs incorporate the formulas and algorithms for calculating g, q, ψ_b, ψ_a, R_{red}, H_p, ψ_m, σ_m, $\Delta\rho(h_{res})$, etc. One program includes formulas that make it possible to find, upon each passage of the cylinder over the soil, $h(\psi)$, $\varepsilon(\psi)$, and $\sigma(\psi)$ at values of ψ equal to ψ_a, $\psi_a/2$, 0, ψ_m, $\psi_b/2$, and ψ_b. The $\sigma(\psi)$ diagrams and the $\sigma(\varepsilon)$ curves plotted according to these data characterize the rheological properties of the soil.

The initial quantities for each calculation are: ρ_{01}, k_1, ρ_{02}, k_2, H_1, H, w, R, L, G, v_{ax}, δ, μ, and f and the coefficients of the correlation dependences $g = g(\rho, \omega, w)$ and $q = q(\rho, \omega, w)$. If, for the cylinder rolling on the soil, a pneumatic-tire wheel is approximately taken, then in the calculations we additionally give the air pressure p_{tire} in the tire, the saturation coefficient of the tread design, and other parameters of the tire, as well as the coefficient k_{non}. The latter takes into account the difference of the contact stresses in various noncentral longitudinal sections of the wheel from the stresses in the central longitudinal section. The value of k_{non} largely depends on the soil density, decreasing with its increase. The calculations have revealed that $k_{non} \in$ [0.62, 0.86]; if $\rho(0.05) \geq 1.2$ g/cm^3, then $k_{non} \approx 0.62$.

Field tests were carried out with the aim of investigating the interaction with the sod-podzol slightly loamy soil upon sequential passages over one track of rigid driven wheels of a tensometric trolley [11] and driven and drive wheels with pneumatic tyres of an MTZ-82 tractor [12]. In the experiments, we determined the normal (radial) contact stresses distributed along the contact lines and the soil density and moisture in different layers of the ploughing horizon before and after the passage of each wheel, as well as other quantities.

Normal contact stresses were measured by tensometric pressure gauges mounted in the wheel. The oscillograms showed the recorded diagrams of $\sigma_{rad}(\psi)$ on the rigid wheels of the tensometric trolley and $\sigma_{rad}(\varphi)$ on the elastic wheels of the MTZ-82 tractor, the marks of the lower position of the corresponding gauges, and the time marks at 0.1-sec intervals.

As a result of the replacement, in the calculations by the method proposed by us, of the elastic wheel by a conventional rigid wheel of radius R_{red}, the $\sigma_{rad}(\varphi)$ diagrams were transformed into $\sigma_{rad}(\psi)$ diagrams. The family of contact stress diagrams obtained upon one passage of the wheel represents the results of several parallel experiments on cyclic loading and unloading of the soil; the latter were processed statistically. Empirical regression lines of the stresses of soil compression by the wheel $\sigma(\psi) = \sigma_{rad}(\psi) \cos \psi$ were plotted. Two groups of computer experiments consisting of a number of sets were performed.

Each set of computer experiments represented calculations of the characteristics of the viscoelastic properties of the soil, its stressed-strained state, the soil-compaction indices, and other quantities upon n sequential passages of a cylinder over one track. The data obtained as a result of the calculation for the previous passage served as the initial data for the calculation for each subsequent passage, beginning with the second one. In the calculations for each subsequent passage, as a new reference point of the deforming-soil-layer depth — the quantity $\tilde{y}$ (i.e., as a new surface of the layer), the coordinate $y = h_{res}$ was taken. The soil density at $\tilde{y} = 0$ is equal to

$$\tilde{\rho}_{01} = \rho_{01} + k_1 h_{res} + \Delta\rho\ (h_{res})\ . \tag{17}$$

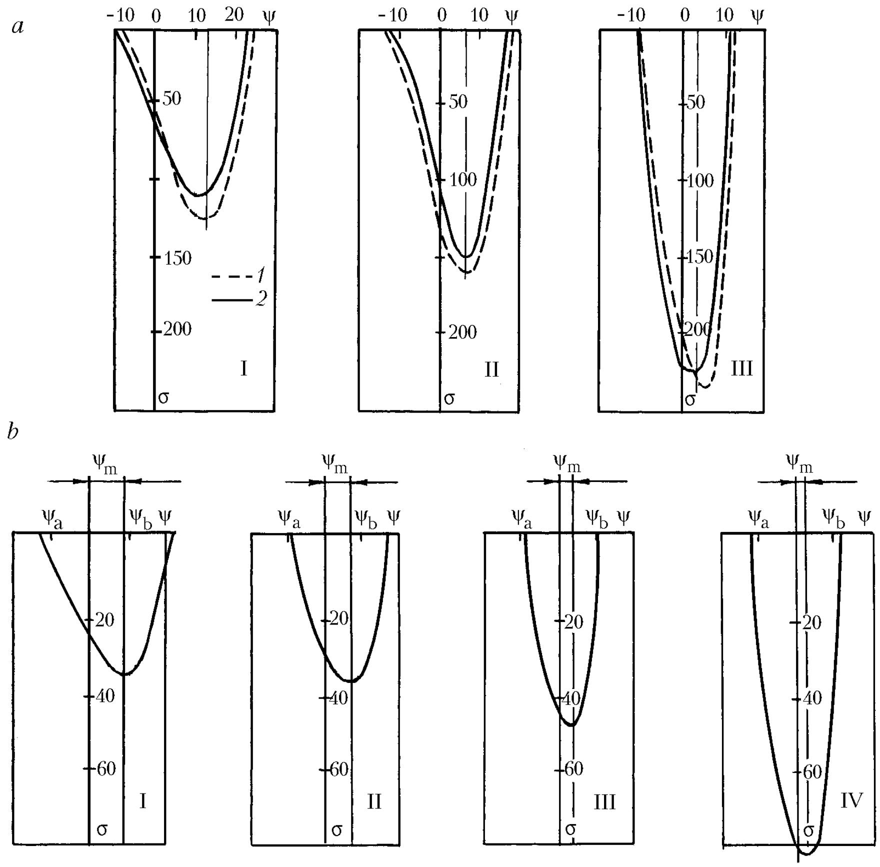

Fig. 2. Diagrams of the soil compression stresses: a) empirical regression lines (1) and theoretical diagrams (2) of the soil compression stresses upon the first (I), second (II), and sixth (III) passages of the front wheel of an MTZ-82 tractor with an 11.2–20 tire over one track (G = 6.08 kN, p_{tire} = 0.11 MPa): I) v_{ax} = 1.17 m/sec; ω = 2.36 sec^{-1}; II) 1.29 m/sec and 2.61 sec^{-1}; III) 1.4 m/sec and 2.85 sec^{-1}; b) calculated diagrams of the soil compression stresses upon the first (I), second (II), tenth (III), and fifty-fifth (IV) passages of a cylindrical roller over one track (G = 5 kN; v_{ax} = 2.1 m/sec; ω = 18.37%). ψ, deg; σ, kPa.

The computer experiments of the first group (12 sets) were performed using the initial test data [12]. We considered the rolling over the soil of the front and rear wheels of an MTZ-82 tractor with 11.2–20 and 16.9R38 tires. The $\sigma(\psi)$ diagrams plotted according to the results of the calculations were compared to their respective empirical regression lines of the experimental diagrams. Figure 2a shows the empirical regression lines and the theoretical $\sigma(\psi)$ diagrams of the stresses of soil compression by a wheel with an 11.2–20 tire upon the first passage over the soil (I) and the second (II) and sixth (III) passages over one track.

The mean deviation of the calculated values of σ_m from the experimental data is 14.2%, the r.m.s. deviation is 11.7%, and for $\tilde{\rho}_{01}$ the r.m.s. deviation is 4.33 and 4.34%, respectively. The proposed computing method can be used to predict the indices of interaction with the soil of a cylinder upon its sequential passages over one track.

In the computer experiments of the second group (14 sets), we investigated the cyclic deformation of the soil under the rolling with slipping of a rigid cylindrical roller. The varying influencing factors were G, v_{ax}, and w, which in different sets of experiments had the following values: G = 2, 5, and 8 kN; v_{ax} = 1, 2.1, and 5 m/sec; w = 14, 16, 18.37, 20, 24, and 26%. Sequential calculations in each of these sets were carried out until an approximate equality $|\psi_a| \approx |\psi_b|$ was attained.

In all sets, calculations upon the first passage of the roller were performed with the following input data: ρ_{01} = 1.1383 g/cm^3, k_1 = 1.7 g/(m·cm^3), ρ_{02} = 1.6228 g/cm^3, k_2 = 0.1737 g/(m·cm^3), H_1 = 0.32 m, H = 1 m, R =

TABLE 1. Characteristics of the Viscoelastic Properties of the Soil, Indices of Its Stressed-Strained State, and Density at a Different Number of Sequential Passages of a Roller over One Track at Different Velocities

Indices	Passage number				
	1	2	3	10	62
q, MPa	2.247 / 4.789	4.556 / 4.940	5.521 / 5.078	9.094 / 5.779	14.978 / 7.833
g	7.072 / 1.065	6.0364 / 0.997	5.604 / 0.935	4.002 / 0.621	0.031 / 0.107
ψ_m, rad	0.2781 / 0.0250	0.1433 / 0.0229	0.1158 / 0.0211	0.0570 / 0.0127	0.0005 / 0.0068
h_{total}, cm	4.18 / 0.95	1.82 / 0.92	1.44 / 0.90	0.79 / 0.78	0.0055 / 0.0201
h_{res}, cm	3.86 / 0.25	1.50 / 0.23	1.12 / 0.21	0.45 / 0.12	0.0005 / 0.0000
k_{rev}	0.0745 / 0.7347	0.1744 / 0.7517	0.2199 / 0.7672	0.4063 / 0.8453	0.9091 / 0.9999
σ_m, kPa	30.54 / 39.39	39.27 / 39.83	42.11 / 40.22	50.69 / 42.10	98.36 / 148.6
$\tilde{\rho}_{01}$, g/cm^3	1.2923 / 1.1484	1.3568 / 1.1576	1.4065 / 1.1660	1.6177 / 1.2093	1.9403 / 1.2931
$\Delta\rho(h_{res})$, g/cm^3	0.088 / 0.057	0.040 / 0.005	0.032 / 0.004	00164 / 0.003	0.0002 / 0.0000
$\tilde{\rho}_1(0.05)$, g/cm^3	1.3727 / 1.2331	1.4351 / 1.2420	1.4831 / 1.2502	1.6863 / 1.2925	1.9995 / 1.3698

Notes: 1. G = 5 kN, w = 18.37%. 2. Upper number — at v_{ax} = 1 m/sec, lower number — at v_{ax} = 5 m/sec.

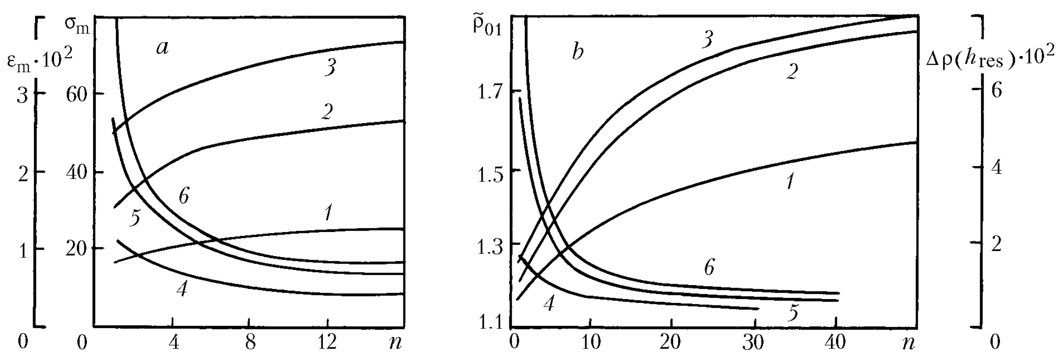

Fig. 3. Dependences on the number of roller passages over one track under various vertical loads on the axis (v_{ax} = 2.1 m/sec, w = 18.37%): a) of the maximal compressive stresses (1–3) and the maximal relative soil compressive deformation (4–6); b) of the density (1–3) and density increment of the soil (4–6) [1 and 4) G = 2; 2 and 5) 5; 3 and 6) 8 kN]. σ_m, kPa; $\tilde{\rho}_{01}$, g/cm^3; $\Delta\rho(h_{res})$, g/cm^3.

0.35 m, L = 1.4 m, δ = 0.1, μ = 0.318, f = 0.32. The characteristics of g and q of the soil were found by the regression equations from [13, 14].

Calculated $\sigma(\psi)$ diagrams for various n were plotted (Fig. 2b).

The change in the characteristics of the viscoelastic properties, the stressed-strained state, and the compaction of the soil with increasing number of n sequential passages of the roller over one track at various G, v_{ax}, and w has been investigated. Table 1 gives the indices found at v_{ax} = 1 and 5 m/sec. Here $h_{total} = R(1 - \cos\psi_b)$, $h_{rev} = R(1 - \cos\psi_a)$, and $k_{rev} = h_{rev}/h_{total}$. Figures 3 and 4 show the curves characterizing the n-dependences at various G

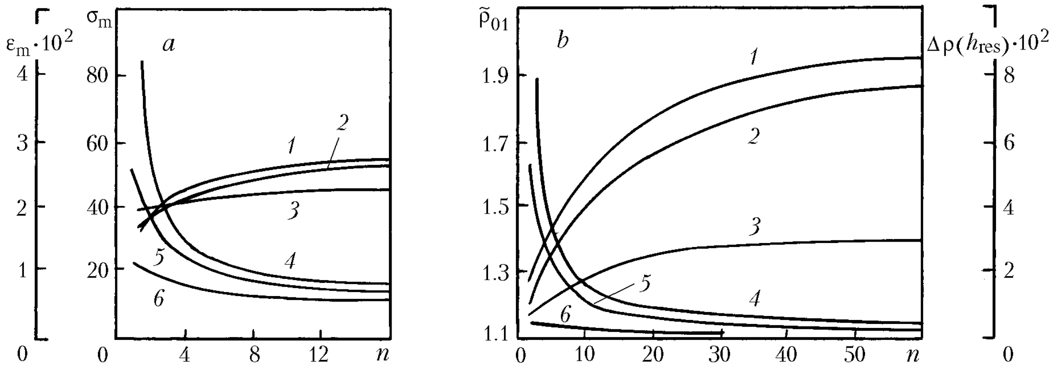

Fig. 4. Dependences on the number of roller passages over one track with various speeds (G = 5 kN, w = 18.37%): a) of the maximal compressive stresses (1–3) and the maximal relative soil compressive deformation (4–6); b) of the density (1–3) and density increment of the soil (4–6) [1 and 4) v_{ax} = 1; 2 and 5) 2.1; 3 and 6) 5 m/sec)]. σ_m, kPa; $\tilde{\rho}_{01}$, g/cm^3; $\Delta\rho(h_{res})$, g/cm^3.

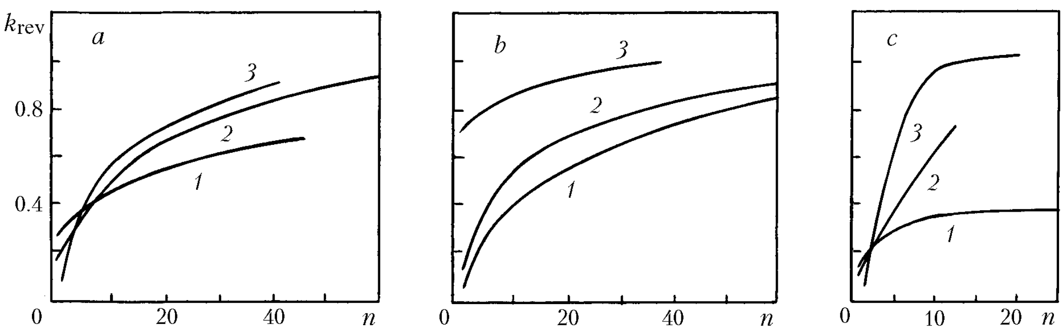

Fig. 5. Change in the share of the reversible deformation of the soil in its total deformation with increasing number of roller passages over one track: a) under various vertical loads on the roller axis [1) G = 2; 2) 5; 3) 8 kN) (v_{ax} = 2.1 m/sec; w = 18.37%)]; b) at various velocities [1) v_{ax} = 1; 2) 2.1; 3) 5 m/sec]; c) at various soil moistures (G = 5 kN; v_{ax} = 2.1 m/sec) [1) w = 16; 2) 20; 3) 24%].

and v_{ax} of the quantities σ_m, ε_m, $\Delta\rho(h_{res})$, and $\tilde{\rho}_{01}$, and Fig. 5 — the dependences of k_{rev} on n at various values of G, v_{ax}, and w.

The calculations have shown that with increasing n the parameters q, k_{rev}, σ_m, and $\tilde{\rho}(\tilde{y})$ increase, with $k_{rev} \to 1$, $\tilde{\rho}(\tilde{y}) \to \rho_{lim}$, and $q \to E$. The values of g, ψ_m, h_{res}, and $\Delta\rho(h_{res})$ decrease and tend to zero. It is seen that the soil properties tend to elastic ones with increasing n. Upon the 62nd passage corresponding to the conditions G = 5 kN, v_{ax} = 5 m/sec, and w = 18.37%, the settlement h_{rev} is very close to h_{total}, i.e., the soil becomes practically elastic (see Table 1). The soil properties change most intensively upon the first four passages (in a number of cases — from 1 to 10 passages); then the process is markedly retarded.

In laboratory and field experiments, unlike computer ones, it would be very difficult (practically impossible) to see how a viscoelastic soil changes into a rigid one with increasing n. This is due to the fact that at large values of the number of cylinder passages over a track the change in all indices with increasing n occurs slowly and it is difficult to detect the difference between the investigated indices at close values of n. Moreover, the necessary number of natural experiments would be very large.

The influence of G, v_{ax}, and w on the considered indices has been investigated. Upon the first passage of the roller over the soil and upon several subsequent passages, an increase in G leads to a decrease in k_{rev}. At large G, however, as n increases, the soil is compacted more intensively than at small values. Therefore, k_{rev} increases with increasing n faster (Fig. 5a). Likewise, k_{rev} changes with increasing w (Fig. 5c).

As G increases, there is an increase in h_{total}, h_{res}, $\Delta\rho(h_{res})$, and σ_m. An increase in G from 2 to 8 kN leads, upon the first passage of the roller over the soil with velocity v_{ax} = 2.1 m/sec, to an increase in h_{total}, h_{res}, $\Delta\rho(h_{res})$, and σ_m by a factor of 3.5, 4.2, 4.1, and 2.5, respectively, and a decrease in k_{rev} under these conditions by a factor of 2.2.

An increase in v_{ax} causes a decrease in h_{total}, h_{res}, and $\Delta\rho(h_{res})$. As v_{ax} increases from 1 to 5 m/sec at G = 5 kN and w = 18.37% upon the first passage of the roller over the soil, h_{total}, h_{res}, and $\Delta\rho(h_{res})$ decrease, respectively, by 71.1, 93.4, and 36.4%, σ_m increases by 29%, and k_{res} increases by a factor of 9.86. The correctional dependence of k_{rev} on v_{ax} upon the first passage of the roller is as follows: $k_{rev} = 0.1388v_{ax} - 0.0175$; the correlation coefficient r = 0.9773. The larger the value of v_{ax}, the faster the approach of k_{rev} to unity with increasing n (Fig. 5b).

An increase in w leads to an increase in h_{total}, h_{res}, and $\Delta\rho(h_{res})$ and a decrease in σ_m. An increase in w from 14 to 26% at G = 5 kN, v_{ax} = 2.1 m/sec upon the first passage of the roller over the soil causes an increase in h_{total}, h_{res}, and $\Delta\rho(h_{res})$ by 177.7, 225.0, and 282.9%, respectively, and σ_m thereby decreases by 27.9%. The correlational dependence of k_{rev} on w at n = 1 is as follows: $k_{rev} = 0.2993 - 0.0074w$, $r = -0.9979$.

CONCLUSIONS

1. Approximate analytical solutions of problems on the rolling with sliding (or with slipping) of circular elastic and rigid cylinders with the formation of a deep track over a viscoelastic base (in this particular case — soil) with a depth-varying density and rheological properties have been obtained.

2. Mathematical modeling of the physical processes proceeding in a viscoelastic soil under its cyclic deformation by the harmonic law realized upon sequential passages of a rolling cylinder over one track has been performed.

3. A method for computing the characteristics of the viscoelastic properties of the soil, the compression stresses, the compressive deformations of the soil, the propagation depth of the soil compressive deformation, the increment of the soil density at various depths, and other indices upon a number of sequential passages of a rolling cylinder over the soil has been proposed.

4. A procedure for calculating the indices being investigated by the proposed method and computer programs for its realization have been developed.

5. Using the programs developed, calculations of the indices of the interaction between the tractor wheels and a sod-podzol slightly loamy soil as they pass sequentially over one track have been made. The calculations were based on the initial data of the field tests.

6. The results of calculating the indices of the interaction with the soil of tractor wheels are in agreement with the experimental data. Therefore, the proposed computing method can be used to predict the indices characterizing the viscoelastic properties, the stressed-strained state, and the compaction of the soil upon a number of sequential passages of circular cylinders (in this particular case — rollers and wheels) over one track.

7. The computer experiments have yielded the characteristics of the viscoelastic properties of the investigated soil, the compression stress and the deformation of the soil, its density increment, and other indices reflecting the soil properties varying upon sequential passages over one track of a roller at various values of the soil moisture, the vertical load on the axis, and the roller velocity.

8. Calculations by the proposed method allow one to predict the change in the soil properties upon passages of a rolling cylinder over one track and to estimate quantitatively the influence of G, v_{ax}, and w on the change in the rheological properties, the stressed-strained state, and the compaction of the soil.

9. It has been shown that after a certain number of passages of a cylinder (depending on the initial values of the soil density and moisture and on the G and v_{ax} values) the initially viscoelastic soil becomes practically rigid. The indices characterizing the properties of the rigid soil have been found.

NOTATION

b_i, $\tilde{c}_i$, coefficients of y in linear dependences $q(y)$, $p(y)$; $C_j(t)$ and $\tilde{C}_j$, coefficients in formula (15) and in the expression for $v(y, \infty)$; E, elastic modulus of the soil, MPa; e_m, maximal radial deformation of an elastic cylinder, cm; F, horizontal force applied to the cylinder axis, kN; f, sliding friction coefficient between the cylinder and the soil; G, vertical force applied to the cylinder axis, kN; g, transformed dimensionless characteristic of the viscoelastic properties of the soil; H and H_1, penetration depth of the deforming layer of the soil before its loading and depth of its top portion, m; H_p, actual depth of propagation of the soil compressive deformation, m; $H_{t.p}$, theoretically possible propagation depth of the soil compressive deformation at $H \to \infty$, m; $h(\psi)$, absolute soil compressive deformation by a cylinder at contact surface points, cm; h_{rev}, h_{res}, and h_{total}, reversible and residual compressive deformation (settlement) of the soil upon passage of a cylinder and total compressive deformation of the soil, cm; k_1, k_2, angular coefficients of the straight lines in dependence (2), g/(cm^3·m); k_{non}, coefficient of stress-distribution nonuniformity on the contact surface of a wheel with a pneumatic tire; k_{rev}, share of reversible deformation of the soil in its total compressive deformation; L, cylinder length, m; M, torque (driving or braking), kN·m; N, resultant of vertical elementary reactions of the soil distributed over the contact surface, kN; n, number of sequential passages of a cylinder over one track; p, parameter of the constitutive differential equation (1) for the soil (characteristic of the viscoelastic properties of the soil), sec^{-1}; p_m, air pressure in the pneumatic tire of an elastic wheel, MPa; q, parameter of the constitutive differential equation (1) for the soil (characteristic of the viscoelastic properties of the soil), MPa; R, actual radius of a cylinder (rigid or elastic), m; R_{red}, radius of the conventional rigid cylinder, by the AKB arc of whose circle the contact line of the rigid cylinder with the soil has been approximated, m; r, correlation coefficient; $r_1(y)$ and r_2(y), functions characterizing the compression-wave damping with depth; T, resultant of the horizontal elementary reactions of the soil distributed over the contact surface, kN; t, time, sec; t_1, time of the cylinder contact with the soil in one axial rotation of it, sec; $v(y, t)$, vertical shifts of the soil, cm; $v_{st}(y)$, stabilized shifts of the soil, cm; v_{ax}, axial velocity of the cylinder, m/sec; w, weight (absolute) moisture of the soil, %; y and $\tilde{y}$, vertical coordinates (depth) of particles of the deforming soil layer before the first loading and before the following cycle of soil deformation, m; α_0, angle characterizing the position of the point B at which the cylinder comes into contact with the soil, rad; $\Delta\rho$, increment of the soil density, g/cm^3; $\Delta\rho(h_{res})$, increment of the initial density of the soil at depth h_{res}, g/cm^3; δ, braking coefficient; ε, relative compressive soil deformation; ε_m, maximum relative compressive soil deformation; μ, lateral expansion coefficient of the soil; ρ, soil density, g/cm^3; ρ_{01} and ρ_{02}, segments cut off on the ρ axis by the straight lines $\rho(y) = \rho_{01} + k_1 y$ and $\rho(y) = \rho_{02} + k_2 y$, g/cm^3; $\tilde{\rho}_{01}$, soil density at $\tilde{y} = 0$, g/cm^3; $\tilde{\rho}(\tilde{y})$, soil density at various depths y upon the passage of a cylinder; $\tilde{\rho}_1(0.05)$, soil density at $\tilde{y} = 0.05$, g/cm^3; ρ_{total}, highest possible density of a soil of undestructed structure, g/cm^3; σ, compressive stress, MPa; σ_m, maximum soil compression stress, kPa; σ_{str}, ultimate strength of the soil, MPa; $\sigma_{rad}(\varphi)$ and $\sigma_{rad}(\psi)$, radial compressive stresses on the contact surfaces of an elastic and a rigid cylinder, MPa; φ_b and φ_a, striking angle and stalling angle of an elastic cylinder, rad; $\varphi(t)$ (at $t \in [0, t_1]$), current angle of elastic cylinder–soil contact, rad; $\tilde{\varphi}_1(y)$, $\tilde{\varphi}_2(y)$, $\tilde{\varphi}_3(y)$, functions determined from (11) at $t = t_1$; ψ_b and ψ_a, striking angle and stalling angle of a rigid cylinder, rad; $\psi(t)$ (at $t \in [0, t_1]$), current angle of elastic cylinder–soil contact, rad; ψ_m, contact angle at which compression stresses acquire the maximum value, rad; ω, frequency of the harmonic process of deformation (angular velocity of a cylinder of radius R), sec^{-1}; ω_{red}, angular velocity of a conventional rigid cylinder of radius R_{red}, sec^{-1}. Subscripts: 01 and 02, numbers of soil density values in dependence (2) at $y = 0$ and $y = H_1$; 1 and 2, numbers of the functions $r_1(y)$ and $r_2(y)$ in formula (11) characterizing the depth-damping of the soil compression wave; 1, 2, and 3, numbers of the functions $\tilde{\varphi}_1(y)$, $\tilde{\varphi}_2(y)$, and $\tilde{\varphi}_3(y)$ in conditions (13); b and a, indices of the striking and stalling angles of a cylinder (corresponding to points B and A in Fig. 1); i, number of the linear portion in dependence (2) ($i = 1$ at $y \in [0, H_1]$, $i = 2$ at $y \in [H_1, H]$); m, maximum value; j, summation index in formula (5) (collocation point number); s, number of collocation points; t, t^2, and t^3, first, second, and third time derivatives; t^3y, ty^3, t^2y, ty, mixed partial time and depth derivatives; y, first depth derivative; non, nonuniformity (of the distribution of stresses); ax, cylinder axis; rev, reversible; res, residual (deformation); red, reduced; total, total (deformation); lim, limited value; str, strength; p, propagation (of deformation); rad, radial (stresses); st, stabilized (shifts and deformations); t.p, theoretical value of the depth of deformation propagation; tire, tire; dashes, derivative numbers.

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